PROGRESS REPORT No.3

Theory of differential equations and their relationship to dynamical systems theory

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I. INTRODUCTION

This report summarizes the research for the period October 1, 1965 through March 31, 1966 of all members of the Center for Dynamical Systems, even though some of the research was not directly supported by your office. Those members of the staff whose work has not received support from this grant are indicated by an asterisk in the list of staff members on the next page (Section II).

Section III is a report on the progress of our research. Section IV lists publications: papers which have appeared or which have been accepted for publication during the period of this report. Section V is a list of visiting lecturers.

II. STAFF MEMBERS 1965-66

*Jose Arraut, Research Associate

*J. Billotti, Research Assistant

N. Chafee, Research Assistant

*M. A. Feldstein, Assistant Professor

J. J. Florentin, Associate Professor

*Robert Grafton, Research Assistant

Jack K. Hale, Professor

*James Heller, Postdoctoral Fellow[1]

Henry Hermes, Assistant Professor

*E. F. Infante, Research Associate

H. J. Kushner, Associate Professor

J. P. LaSalle, Professor

Solomon Lefschetz, Visiting Professor

*Marshall Leitman, Research Associate

*Jorge Lewowicz, Research Assistant

Kenneth R. Meyer, Assistant Professor

*Z. Opial, Visiting Assistant Professor[2]

*A. E. Pearson, Assistant Professor

M. M. Peixoto, Associate Professor

Carlos Perelló, Research Associate

*P. R. Sethna, Visiting Professor[3]

Leonard Weiss, Associate Professor

F. Wesley Wilson, Research Associate

W. M. Wonham, Associate Professor

1 Canadian National Research Council Post Ph.D. Fellowship

2 On leave from Jagellouian University, Krakow

3 On leave from the University of Minnesota

*Did not receive support from this Grant.

III. RESEARCH RESULTS

A. Research Completed

1. Control theory - Feedback control

Hermes has submitted a paper on "Discontinuous vector fields and feedback control" to the Proceedings of the Symposium on Differential Equations and Dynamical Systems, Puerto Rico, 1965 (to be published by Academic Press).

2. Functional differential equations

a. Linear functional differential equations

Meyer has studied linear functional differential equations from the viewpoint of functional analysis. With this more general viewpoint he has been able to prove the analog of the Jordan normal form theorem and derive an expression for solution to the nonhomogeneous equation similar to the variation of constants formula for ordinary differential equations. It is hoped that this more general approach will help clarify some of the basic concepts and lead to a deeper understanding of such equations.

b. Oscillations

Perelló has submitted a paper on "Periodic solutions of differential equations with time lag containing a small parameter" to the Journal of Differential Equations.

c. Bifurcation problems

Chafee has completed the writing of his Ph.D. dissertation entitled "Two bifurcation problems".

3. Graph and network theory - Basic theory

Lefschetz has organized the material mentioned in the previous report and written it up from top to bottom in good form so that at the present time it may form the skeleton of a small contribution called "Topology for Graph Specialists with Some Applications". The organizing of this material turned out to be much more difficult than first expected.

4. Qualitative theory of ordinary differential equations

a. Structural stability

A simplified and more clear-cut proof of an approximation theorem of Kupka and Smale has been given by Peixoto. A paper on this subject is to appear in the Journal of Differential Equations. Peixoto has also completed a paper on "Qualitative theory of differential equations and structural stability" which is to appear in the Proceedings of the Symposium on Differential Equations and Dynamical Systems, Puerto Rico 1965 (Academic Press).

b. Boundary value problems for ordinary nonlinear differential equations

Two manuscripts have been written by Opial in this period: (1) "On the existence of solutions of linear problems for ordinary differential equations" (with A. Lasota), submitted for publication in Bull. Acad. Polon. Sci., Série des sci. math., astr. et phys.; and (2) "On a theorem of O. Aramă", which has been prepared for publication.

c. Relative invariant sets

Lewowicz has completed research on relative invariant sets and has prepared a Ph.D. dissertation on this subject.

5. Stability of dynamical systems

Stability theory.

LaSalle has completed a unified development and extension of classical Liapunov theory which generalizes the notion of a Liapunov function and shows precisely the asymptotic information provided by a Liapunov function. Several accounts of this research will appear in proceedings of meetings and a CDS report has been prepared. All of these simply describe and illustrate the theory and a paper complete with proofs is being prepared.

6. Stochastic dynamical systems

a. Stochastic linearization

A computer algorithm was designed by Wonham (and successfully tested) to perform a stochastical linearization analysis of second and third order nonlinear control systems. This method provides quick, order-of-magnitude estimates of statistical moments in stochastic systems, and is practically much more effective than the Liapunov (or comparison function) method he developed last year. The method of statistical linearization is of course not new, but its "computerized" version is both novel and of practical interest. When more experience with it has been gained the results will be reported formally. A graduate student is studying the convergence problem theoretically. A criterion is being sought which would specify a class of nonlinear systems for which one could guarantee a priori that the algorithm (a computation by successive approximations) would converge. This study is of theoretical interest, but a convergence criterion of any scope would probably be more difficult (and expensive) to apply than the algorithm itself, regarded as a computer experiment.

Additional results have been obtained in connection with a stochastic stability problem which arose in connection with Wonham's consulting work at NASA ERC, Cambridge. The results are capable of further extension and publication is not contemplated at present.

In exploring the role of statistical linearization techniques in the state estimation procedure, Sunahara has established a theoretical basis for these techniques. By using this statistical linearization technique the state estimation problems for nonlinear dynamical systems have been solved by Sunahara.

b. Stochastic stability

Kushner has generalized some of his previous results (e.g., in CDS Report 65-1). He now has analogs of the usual Liapunov theorems which are valid for arbitrary right continuous strong Markov processes. Given that a function, which is in the domain of the weak infinitesimal operator of a stopped process (of the original process), satisfies certain conditions (analogs to these usually imposed on Liapunov functions), then one may make inferences regarding the boundedness or asymptotic behavior of the sample paths of the process. The theorems are too numerous to mention here. There are also analogs of the theorems on sufficient conditions for long-range stability (no finite escape time) and there are analogs of Yoshizawa's theorems on equistability, etc.

c. Stochastic control

Kushner has given, as far as we know, the first mathematical proof of the optimality of the usual solution to the linear system - white noise driven - quadratic cost - fixed time problem - both with and without white noise corrupted observations. A specific class of comparison controls with respect to which the solution is optimal is the class of either (a) locally Lipschitz controls or (b) bounded non-anticipative controls.

The result is certainly not unexpected. Nevertheless all prior proofs have been by formally using dynamic programming and obtaining one solution to the degenerate parobolic equation which arises. The result is an example of some general results on sufficient conditions for the optimality of a control.

d. Finite time stability and first exit times

Kushner has extended his results on finite time stability and first exit times and some estimates have been strengthened. In particular, (omitting details) let $\tilde{\mathcal{A}}$ be the weak infinitesimal operator of a process with $V \geq 0$ and V in the domain of $\tilde{\mathcal{A}}$. Then, if

$$\tilde{\mathcal{A}} V \leq -\mu V + \varphi_t , \quad \mu \geq 0 \quad , \quad \varphi_t \geq 0$$

$$m \geq \max_{t \leq t} \varphi_t / u \quad ,$$

with x as the initial condition of the process,

$$P_x \{\sup_{T \geq t \geq 0} V(x_t) \geq m\} \leq 1 - (1 - \frac{V(x)}{m}) e^{-\Phi_T/m}$$

where

$$\Phi_T = \int_0^T \varphi_t dt \quad .$$

These estimates have also been used in the design of controls for systems satisfying given specifications on finite time stability.

B. Continuing Research

1. Control theory.

Attainable sets. Hermes in his study of functional equations for attainable sets has narrowed the problem to the study of attainable sets which arise from a particular class of contingent equations. These do include, however, time optimal control problems for which known existence conditions are satisfied.

An analytic method for computing a set valued function $S(t)$ which contains the boundary of the attainable set $\mathcal{A}(t)$, has been developed by Hermes. He has shown, under appropriate conditions, that if t_o is the initial time, then for $t-t_o$ positive but sufficiently small, $S(t) = \partial\mathcal{A}(t)$. ($\partial\mathcal{A}(t)$ denotes boundary of $\mathcal{A}(t)$). For arbitrary $t - t_o$, one still has $\partial\mathcal{A}(t) \subset S(t)$. Further properties of $\mathcal{A}(t)$ can also be inferred from $S(t)$.

2. Functional differential equations

a. Functional differential equations of neutral type

Hale and Perelló are continuing their work on the extension of the results of Hale (Contr. to Diff. Eqns., Vol. 2 (1963)) to functional differential equations of neutral type. They consider a functional differential equation of the form

(1) $$\dot{x}(t) = f(x_t) + g(\dot{x}_t) , \ t \geq 0 ,$$

where f and g are bounded linear functionals on $C[-r, 0]$, and the measure associated with g is nonatomic at 0.

To this equation is associated the one-parameter semigroup of operators $T(t)$ defined by $T(t)\varphi = x_t(\varphi)$, where $x_t(\varphi)$ is the unique solution of (1) with initial condition φ. This semigroup has an infinitesimal generator $\mathcal{A}$ which has only point spectrum which is discrete and the real part of its elements has an upper bound. They are trying now to prove that if all the spectrum has real part less than γ then for any $\beta > 0$

$$\|T(t)\varphi\| \leq Ke^{(\gamma+\beta)t}\|\varphi\| \quad \text{for some } K \text{ and for all } t \geq 0.$$

In particular this would establish asymptotic stability of the 0 solution if the spectrum has all negative real parts and is bounded away from the imaginary axis.

b. Numerical methods

Feldstein has developed an alternate approach to the numerical solution of retarded ordinary differential equations. There is promise of being able to attain higher order methods in a simple manner.

c. Oscillations

A geometric method has been developed and used to show the existence of a periodic solution of the equation

$$\dot{x}(t) = -\alpha x(t-1)[1 + x(t)] \qquad \alpha > \pi/2 .$$

(The same result was obtained by G. S. Jones through analysis of the equation.) The geometric method is being generalized so that it may be applied to any functional differential equation of the form: $\dot{x}(t) = L(x_t) + N(x_t)$ where L is a linear operator and N is a nonlinear operator satisfying a Lipschitz condition. This has been done by Grafton, under the direction of Hale.

3. Graph and network theory

Applications. Weiss is currently working with Lefschetz on applications of algebraic topology to problems in graph theory and network theory. One problem in which the use of certain tools of algebraic topology appear to arise quite naturally is the problem of existence and uniqueness of solutions to linear electrical networks. The question is posed as follows: Given an arbitrary linear electrical network and an arbitrary set of external voltages and currents impressed on the network, what are the necessary and sufficient conditions on the network for existence of a unique solution to the network equations (i.e., existence of a unique set of branch voltages and branch currents satisfying Kirchoff's voltage and current laws respectively)?

4. Partial differential equations

Stability theory

The program of attempting to carry over to partial differential equations the general Liapunov theory of LaSalle is being continued. A seminar has been organized to study stability problems arising in partial differential equations and some special problems are being investigated.

5. Qualitative theory of ordinary differential equations

Liapunov stability theory

Wilson has been investigating converse Liapunov theorems for asymptotically stable systems, with particular interest in (1) smoothing existing Liapunov functions; and (2) generating Liapunov functions for systems which have noncompact sets for their attractors. Wilson's paper "On the minimal sets of nonsingular flows" has been revised and shortened as suggested by the referee and resubmitted to the Annals of Mathematics. A paper on "The structure of the level surfaces of a Liapunov function" has been rewritten by Wilson with extensive changes in the last section and resubmitted to the Journal of Differential Equations.

6. Stability of dynamical systems

Generation of Liapunov functions

Work on linear and nonlinear timevarying systems is continuing. In previously reported work Infante developed a simple technique for the generation of quadratic Liapunov functions for linear systems. Surprisingly, the Liapunov functions obtained in this manner are the best possible quadratic functions for certain types of equations. Research is being continued on this problem with the goal of determining a method for the generation of the best quadratic Liapunov function for a given equation. This problem is strongly related to a generalization of the circle criterion for the stability of time-varying systems.

7. Stochastic dynamical systems

Stochastic control

Research is continuing by Wonham on the theory of stochastic control and the results are being reported in preliminary form in Wonham's course notes for a seminar this semester at M.I.T. Much of this research amounts to filling in technical details connected with the dynamic programming formulation of stochastic control problems. As such, it is of somewhat minor importance compared to the numerical work described above. A full account is being prepared for a short monograph, to be published by Academic Press, in a volume edited by A. T. Bharucha-Reid. This work should be completed by July 1.

A theoretical technique for determining the optimal final-value control strategy with the help of estimated information concerning the system state is now under study by Sunahara. The situation considered is one in which a dynamical system is described by nonlinear vector difference equations of the following form:

$$x(k+1) = f\,[x(k),k] + G(k)\,u(k) + w(k)$$

and the observed signal $Z(k)$ is expressed by

$$Z(k) = h\,[x(k),k] + n(k)$$

where

x: the (n x 1) state vector of the system

u: the (m x 1) vector, the input to the system (m = n)

w: random noise applied to the system

G: (n x m) matrix

v: vector representing the measurement noise

f and h: vector-valued nonlinear functions respectively.

The optimal synthesis of such a dynamical system depends upon the solution of two problems. The first is the extraction of all possible useful information from measurable quantities $Z(0), \cdots, Z(k-1)$ and $Z(k)$ in the dynamical system to be controlled. This problem has been solved by the application of estimation theory and an extensive use of the statistical linearization technique. The derivation of a control strategy, which is the second problem, is established by making use of the information provided by estimation to produce system input signals which minimize the specified performance index. The digital simulation studies are also being initiated, with the development of a signal flow chart for a computer algorithm.

8. System identification

For the past three months Pearson has focused his attention on the system identification problem, particularly the identification of the kernel function matrix which characterizes the input-output relation of a linear system. The theoretical aspects of the approach were reported in a paper presented at the 1965 JACC, and subsequently extended in a paper to be presented at the Third IFAC Congress. The research effort for the past three months has been directed toward the practical aspects of implementing the proposed approach. Using elements of the calculus of operators in normed linear spaces, error analyses have been carried out to compare the convergence properties of various iterative techniques (generalized Newton and steepest descent) for constructing a sequence of kernel function matrices relative to the identification problem. A computer program has been written by a graduate student advisee to investigate the convergence properties of one of the generalized Newton iterations in application to identifying single input-output systems. Plans are underway to write programs for multivariable systems using both the steepest descent and generalized Newton iterations.

C. New Research

1. Control theory

Stabilization and performance criteria

A study of the stabilization of nonlinear systems and its relation to optimal control is being started under the direction of LaSalle. A few examples illustrate there is a problem here and that the sense in which a stabilized system is optimal is related to the problem of selecting good performance criteria. Examples also indicate that if one wants a "strong" stability then the performance criterion cannot be selected a priori but will depend on the system to be controlled. This problem is being assigned to a graduate student as a possible Ph.D. dissertation.

2. Functional differential equations

Numerical methods

Feldstein has looked at the problem of determining the zeros of functions defined as the solution of certain retarded ordinary differential equations. A few interesting facts have been proven. A computer study is to be carried out. The programming is to be done by an undergraduate student under Feldstein's direction as a term project.

3. Graph and network theory

Identification of networks

Identification of the network as a 1-dimensional complex allows the above problem to be formulated in terms of finding appropriate properties of transformations on groups; in particular one must investigate a certain transformation which takes the group of 1-chains of the network into the group of 1-cochains. Preliminary results indicate that this approach is not only useful in solving the originally posed problem but may shed some light on such difficult questions as the nature of "duality" among nonplanar networks. This is being done by Weiss.

4. Partial differential equations

Theory of oscillations

Hale has begun a discussion of oscillatory properties in partial differential equations with a small parameter. The first step has been an investigation of the nonlinear nonautonomous wave equation when there is complete resonance between the forcing terms and the free vibrations of the string. This phase of the work is nearing completion and it has been shown that the method used by Cesari and Hale for similar problems in ordinary differential equations can be carried over to this type of partial differential equation to obtain necessary and sufficient conditions for the existence of periodic solutions. These conditions are given in terms of a set of bifurcation equations for a solution of the homogeneous wave equation. It is hoped that these results can be extended to cases of "near" resonance and autonomous problems. The ideas are exactly the same as with ordinary differential equations but difficulties arise because of a loss of derivatives in the successive approximations.

5. Qualitative theory of ordinary differential equations

a. Systems with first integrals

A conjecture by Peixoto to the effect that every system with one first integral can be approximated by a structurally stable system has been reduced by Peixoto to the simpler problem of showing that a certain gradient-like system is structurally stable. A statement of Smale ("Dynamical Systems", Boll. Soc. Mat. Mexicana, 1960) implies that this conjecture is true for $n = 3$. For $n > 3$ there are technical difficulties but Peixoto feels there is little room for doubting that it is true and believes the technical difficulties can be overcome and a proof can be given.

b. Structural stability

On a compact manifold M^2 let Σ be the set of structurally stable systems and $X \in \Sigma$. Then the fundamental group of Σ at X can be computed explicitly once the relative position of closed orbits and singularities of X are given; it is always finitely generated. This is being done by Peixoto.

c. Local first integrals

Arraut is considering a system of equations

$$(*) \qquad \dot{x} = X(x) \quad , \quad x = (x^1, \cdot \cdot, x^n)$$

under the assumption that (*) has a singularity at the origin. The problem is to find the maximum number of independent local first integrals around the origin which the system (*) admits. Conversely, if the system (*) admits $k, 1 \leq k \leq n-1$ local first integrals around the origin, what can we say about the origin?

d. Boundary value problems for ordinary nonlinear differential equations

Since joining the Center in February Opial has continued his research on extending the theory of linear problems to nonlinear ordinary differential equations (existence of solutions, continuous dependence on right-hand sides and boundary value conditions).

e. Stability of Hamiltonian systems

Meyer has recently become interested in classical mechanics and Hamiltonian systems. In preparation for a study of stability of solutions to conservative systems he has organized a seminar to study the papers of the Russian mathematician Arnold. Several members of the Center for Dynamical Systems have taken part in the seminar.

6. Stability of dynamical systems

Stability of bang-bang control systems and surge tanks

At the suggestion of Lefschetz, Infante is studying the problem of the generation of Liapunov functions for two particular classes of systems. One class consists of Bang-Bang control systems, and it is desired to determine for what kind of switching surfaces stability of the origin can be guaranteed. The second type of problem is related to compound surge-tanks, extending previous work for simple surge-tanks already reported.

7. Stochastic dynamical systems

a. Stochastic control

A computer program is being written under Wonham's supervision to calculate solutions of a partial differential equation which arises in stochastic control problems. The equation is

$$\mathcal{L}v(x) = \lambda - L(x) \tag{1}$$

where $\mathcal{L}$ is a linear elliptic equation with variable coefficients (the differential generator of a diffusion process), L is a function which measures the cost of control in a state x, and λ is the expected value of L relative to the ergodic measure of the process.

Eq. (1) is solved numerically by expanding the unknown function v in a series of multinomials in the state variables $x_1, x_2, \ldots$ and determining optimum values of the expansion coefficients by a least squares procedure. The program is a rather sophisticated symbol-manipulation (as distinct from numerical) algorithm and is being written by a graduate student of the Division of Applied Mathematics. At this date the program is nearly complete. Runs will begin shortly to determine the performance of nonlinear second-order control systems subjected to random noise. The numerical method has been designed with a view to applying Bellman's procedure of approximation in policy space to determine optimum control laws. The ultimate goal of this research is to determine whether, in typical low order systems, optimal stochastic control leads to systems which are appreciably better than systems designed by the usual deterministic criteria.

b. Evaluation of optimal control system performance

Sunahara has begun a study of the evaluation of the dynamic behavior including stochastic stability of the performance of optimal control systems. A simplified configuration of the optimal control system is utilized.

IV. PUBLICATIONS

Arraut, Jose-Luis

"Note on structural stability", to appear, Bull. of Amer. Math. Soc.

Hale, Jack K.

"Averaging methods for differential equations with retarded arguments and a small parameter", J.Diff.Eq. 2, No. 1 (1966) 57-73

"Geometric theory of functional differential equations", to appear, Proc. of Int'l Symposium on Diff. Eq. and Dynamical Systems, Puerto Rico, 1965, (Academic Press).

Hermes, Henry

"Sufficiency in linear time optimal control", to appear J. Math. Anal. and Appl.

"Discontinuous vector fields and feedback control", to appear, Proc.,Int'l. Symp. on Diff.Eq. and Dynamical Systems, Puerto Rico, 1965.

Infante, E. F.

"The simulation of the differential equations of a simple surge-tank", L'Energia Elettrica (Milan, Italy) 17 (1965) 520-27.

"On the stability of the oscillations of a simple surge-tank" (with L.G. Clark), Trans. ASME Ser. E (J. Appl. Mech.) 32 (1965) 945-947

"Finite time stability under perturbing forces and on product spaces" (with L. Weiss), to appear Proc., Int'l Symp. on Diff. Eq. and Dynamical Systems, Puerto Rico, 1965.

Kushner, Harold J.

"On the construction of stochastic Liapunov functions", IEEE, Trans., G-AC, 10, No. 4 (1965) 477-478

"A note on the maximum sample excursions of stochastic approximation processes", Ann. Math. Statist., April 1966.

"On the status of optimal control and stability for stochastic systems", IEEE, Int'l Conv. Record, Part 6 (1966)

"On the existence of optimal stochastic controls", SIAM J. on Control 3, No. 3 (1965) 463-474

"Sufficient conditions for the optimality of a stochastic control", SIAM J. on Control 3, No. 3 (1965) 499-508

LaSalle, J. P.

"Eventual Stability", Proc. 2nd IFAC Congress, Basle, 1963, Butterworth, London, 1964, Vol. II, 556-560

"Liapunov's Second Method", to appear in Proc. of NATO Summer Institute held Sept. 1965, Padua, Italy.

"Stability theory and the asymptotic behavior of dynamical systems", to appear Proc., Int'l Conf. on Dynamic Stability of Structures, Evanston, Ill., October, 1965. (Pergamon Press)

"An invariance principle in the theory of stability", to appear, Proc. Int'l Symp. on Diff.Eq. and Dynamical Systems, Puerto Rico, 1965.

Lefschetz, Solomon

"Geometric differential equations: recent past and proximate future", to appear, Proc.,Int'l Symp. on Diff. Eq. and Dynamical Systems, Puerto Rico, 1965.

Meyer, Kenneth R.

"On the existence of Lyapunov functions for the problem of Lur'e", SIAM J. on Control 3, No. 3 (1965) 373-384.

Opial, Zdzislaw

"On the existence of solutions of linear problems for ordinary differential equations" (with A. Lasota), submitted to Bull. Acad. Polon. Sci., Serie des sci. Math., astr. et phys.

Peixoto, M. M.

"On an approximation theorem of Kupka and Smale", to appear in J. Diff. Eq.

"Qualitative theory of differential equations and structural stability", to appear in Proc., Int'l Symp. on Diff.Eq. and Dynamical Systems, Puerto Rico, 1965.

Perello, Carlos

"Periodic solutions of differential equations with time lag containing a small parameter", submitted to J. Diff. Eq.

"A note on periodic solutions of nonlinear differential equations with time lags", to appear in Proc., Int'l Symp. on Diff Eq. and Dynamical Systems, Puerto Rico, 1965.

Weiss, Leonard

"Finite time stability under perturbing forces and on product spaces" (with E. F. Infante), to appear, Proc., Int'l Symp. on Diff. Eq. and Dynamical Systems, Puerto Rico, 1965.

Wilson, F. Wesley

"On the minimal sets of nonsingular vector fields", to appear, Annals of Math.

V. LECTURES BY VISITORS

Abraham, Ralph Princeton University	March 31	Applications of Transversality
Abramson, Norman Harvard University	Dec. 7, 1965	Some Results and Questions in Pattern Recognition
Dolezal, Vaclav Mathematics Institute, Czechoslovak Academy of Sciences Visiting Prof., SUNY at Stony Brook	March 24	On non-canonic systems of linear integro-differential equations
Kurzweil, Jaroslav Mathematics Institute Czechoslovak Academy of Sciences	Jan. 24 and 26	Topics in Control Theory <u>and</u> Invariant Manifolds for Dynamic Systems
Letov, A. M. Institute of Automation and Remote Control, Moscow	Feb. 18	Recent Advances in Control Theory
Miller, Richard K. University of Minnesota	Feb. 8	Integral Equations
Vorel, Zdenek Mathematics Institute Czechoslovak Academy of Sciences	Apr. 1	Ordinary Differential Equations in Banach Spaces and Applications to Functional Differential Equations